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Interaction of moving interface collinear Griffith cracks under antiplane shear

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Abstract

The interaction between three moving collinear Griffith cracks under antiplane shear stress situated at the interface of an elastic layer overlying a different half plane has been studied. Fourier transform and finite Hilbert transform techniques have been employed to solve the problem. Approximate analytical expressions for stress intensity factors at the crack tips have been derived for large thickness of the elastic layer. Numerical results connected to the interaction effect have also been obtained. Depending on the spacing of the cracks, their common velocity of propagation and the depth of the layer, occurrence of shielding and amplification phenomena of the cracks have been noticed.

Keywords: Moving Griffith crack; Interface; Antiplane shear stress; Integral transform; Integral equation; Stress intensity factors; Stress intensity magnification factors; Interaction

1. Introduction

In the recent past the concerns with mechanical failures initiating largely at the interfacial regions of bonded materials have led to extensive studies for the purpose of understanding the interaction between flaws that may exit in these regions and applied loads and other environmental factors. Mismatch between materials forming composities produces residual stress, which may initiate debonding, delamination and microcracks. Physical existence of a pair of collinear Griffith cracks is a simple example of such flaws. Existence of such flaws has been considered by many authors like Willmore (1949), Lowengrub (1975), Erdogan and Wu (1993), Das (2003), Shbeeb et al. (1999), etc.

The diffraction of elastic waves by one or more cracks moving along the interface of two elastic media has been studied by Dhaliwal et al. (1992a,b), Das and Ghosh (1992), Srivastava et al. (1980), Bostrom (1987), etc. But to our knowledge, the diffraction of elastic waves by three moving interfacial cracks has not been investigated so far. Analytical studies of crack interaction problems can be found in Sneddon and Lowengrub (1969), Rose (1986), Lam et al. (1993), Brencich and Carpinteri (1996), Das and Patra (1996) and many others.

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In the present paper, the interaction between three collinear Griffith cracks under antiplane shear stress, propagating with constant velocity along the interface of an elastic layer of finite thickness h overlying a semi-infinite medium of different elastic properties, has been considered. The resulting mixed boundary value problem is reduced to the solution of a set of triple integral equations, which are further simplified by using a perturbation technique retaining terms up to the order of h^{-2} for large h. The resulting integral equations are then solved using Hilbert transform technique and Cooke's result and approximate analytical expressions for stress intensity factors are obtained. Numerical results for the interaction of outer cracks on the central one and conversely through stress intensity magnification factors have been calculated. Graphical plots of these results confirm the evidence of the phenomena of shielding and amplification of the cracks depending upon spacing of cracks, their common velocity of propagation and the depth of the layer.

2. Statement and formulation of the problem

Consider the interaction of three collinear Griffith cracks situated at the interface of an elastic layer of depth h overlying a different elastic half-plane. Let the cracks be opened under time independent antiplane shear forces and are moving with a constant velocity v. Introducing the index $i = 1, 2$ to represent quantities in the region of the layer and the half plane respectively, the only non-vanishing component of displacement vector in the region are $W^{(i)} = W^{(i)}(X, Y, t)$ in terms of fixed coordinates (X, Y, Z) and they satisfy, in absence of body forces, the following deferential equations of motion:

$$\frac{\partial^2 W^{(i)}}{\partial X^2} + \frac{\partial^2 W^{(i)}}{\partial Y^2} = \frac{1}{k_i^2}\frac{\partial^2 W^{(i)}}{\partial t^2}, \tag{2.1}$$

where $k_i = \sqrt{\frac{\mu_i}{\rho_i}}$, ρ_i and μ_i are shear wave velocity, density and shear moduli respectively of the materials. Introducing Gallelian transformation such as $x = X - vt$, $y = Y$, $z = Z$ and $t = t$, the above Eq. (2.1) becomes

$$s_i^2\frac{\partial^2 \omega^{(i)}}{\partial x^2} + \frac{\partial^2 \omega^{(i)}}{\partial y^2} = 0, \tag{2.2}$$

where

$$s_i^2 = 1 - \frac{v^2}{k_i^2} \quad \text{and} \quad W^i(X, Y, t) \equiv \omega^i(x, y), \tag{2.3}$$

is the displacement independent of time t.

The present study deals with the subsonic propagation $(0 < v < k_i)$ of cracks. So the present analysis is not applicable to the case of supersonic propagation $(v \geqslant k_i)$ of cracks.

The cracks are now defined by $|x| < b$, $y = 0$ and $c < |x| < 1$, $y = 0$ $(b < c)$ (Fig. 1). Eq. (2.2) is to be solved under the boundary conditions

$$\tau_{yz}^{(1)}(x, 0^+) = \tau_{yz}^{(2)}(x, 0^-) = -p(x), \quad |x| < b, c < |x| < 1, \tag{2.4}$$

$$\tau_{yz}^{(1)}(x, 0^+) = \tau_{yz}^{(2)}(x, 0^-), \quad b \leqslant |x| \leqslant c, |x| \geqslant 1, \tag{2.5}$$

$$\omega^{(1)}(x, 0^+) = \omega^{(2)}(x, 0^-), \quad b \leqslant |x| \leqslant c, |x| \geqslant 1, \tag{2.6}$$

$$\text{and } \tau_{yz}^{(1)}(x, h) = 0, \quad -\infty < x < \infty, \tag{2.7}$$

together with

$$\omega^{(i)}(x, y) = 0, \quad (x^2 + y^2)^{1/2} \to \infty,$$

$$\text{and} \quad \tau_{yz}^{(i)}(x, y) = 0, \quad (x^2 + y^2)^{1/2} \to \infty, \quad i = 1, 2,$$

where, $p(x)$ being the applied antiplane shear stress independent of time.

3. Solution of the problem

Employing the Fourier cosine transformation,

$$\overline{\omega}^{(i)}(\xi, y) = \sqrt{\frac{2}{\pi}}\int_0^\infty \omega^i(x, y)\cos\xi x\,\mathrm{d}x \tag{3.1}$$

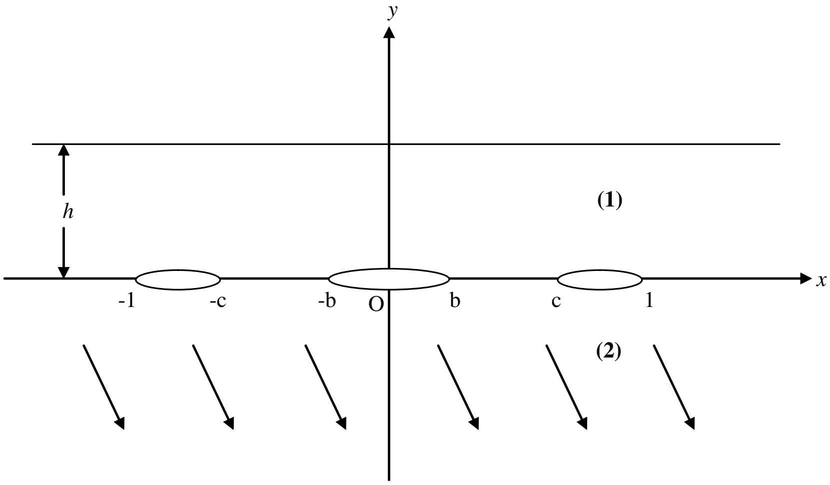

Fig. 1. Geometry of the problem.

the Eq. (2.2) becomes

$$\frac{d^2}{dy^2}\overline{\varpi}^{(i)}(\xi,y) - \xi^2 s_i^2 \overline{\varpi}(\xi,y) = 0. \tag{3.2}$$

The solution of (3.2) for the layer $0 \leqslant y \leqslant h$ is

$$\omega^{(1)}(x,y) = \sqrt{\frac{2}{\pi}}\int_0^\infty \left[A_1^{(1)}(\xi)e^{-s_1\xi y} + A_2^{(1)}(\xi)e^{s_1\xi y}\right]\cos\xi x d\xi, \quad 0 \leqslant y \leqslant h \tag{3.3}$$

and for the half plane $-\infty < y \leqslant 0$ is

$$\omega^{(2)}(x,y) = \sqrt{\frac{2}{\pi}}\int_0^\infty A_1^{(2)}(\xi)e^{s_2\xi y}\cos\xi x d\xi, -\infty < y \leqslant 0 \tag{3.4}$$

and then the non-vanishing stress components are given by

$$\tau_{yz}^{(1)}(x,y) = \sqrt{\frac{2}{\pi}}\mu_1 s_1 \int_0^\infty \xi\left[-A_1^{(1)}(\xi)e^{-s_1\xi y} + A_2^{(1)}(\xi)e^{s_1\xi y}\right]\cos\xi x d\xi, \quad 0 \leqslant y \leqslant h, \tag{3.5}$$

$$\tau_{yz}^{(2)}(x,y) = \sqrt{\frac{2}{\pi}}\mu_2 s_2 \int_0^\infty \xi A_1^{(2)}(\xi)e^{s_2\xi y}\cos\xi x d\xi, \quad -\infty < y \leqslant 0. \tag{3.6}$$

Boundary conditions in Eqs. (2.4), (2.5) and (2.7) give rise to

$$A_1^{(2)}(\xi) = \frac{\mu_1 s_1}{\mu_2 s_2}\left[A_2^{(1)}(\xi) - A_1^{(1)}(\xi)\right] \tag{3.7}$$

and $\quad A_2^{(1)}(\xi) = A_1^{(1)}(\xi)e^{-2\xi h s_1}$ (3.8)

Now setting $\quad f(\xi) = A_1^{(1)}(\xi)\left[1 + \frac{\mu_2 s_2 - \mu_1 s_1}{\mu_2 s_2 + \mu_1 s_1}e^{-2s_1\xi h}\right]$ (3.9)

the boundary conditions in (2.4) and (2.6) finally yield the following triple integral equations

$$\int_0^\infty \xi f(\xi)[1 + M(\xi h)]\cos\xi x d\xi = \sqrt{\frac{\pi}{2}}\frac{p(x)}{\mu_1 s_1}, \quad 0 < x < b, \; c < x < 1, \tag{3.10}$$

$$\int_0^\infty f(\xi)\cos\xi x d\xi = 0, \quad b < x < c, \; x > 1, \tag{3.11}$$

where $M(\xi h) = -(1 - \tanh s_1 h\xi)/(1 + \frac{\mu_1 s_1}{\mu_2 s_2}\tanh s_1 h\xi)$, for the determination of the unknown function $f(\xi)$.

It should be noted that $M(\xi h) \to 0$ as $h \to \infty$, which is obvious as the depth of the layer increases, the effect of the layer boundary diminishes and finally goes to zero.

$$\text{Setting} \quad f(\xi) = \frac{1}{\xi}\int_0^b h(t)\sin(\xi t)\mathrm{d}t + \frac{1}{\xi} int_c^1 g(u^2)\sin(\xi u)\mathrm{d}u, \tag{3.12}$$

it is found that Eq. (3.11) is identically satisfied if

$$\int_c^1 g(u^2)\mathrm{d}u = 0. \tag{3.13}$$

Eq. (3.10) under Eq. (3.12) leads to

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_0^b h(t)\log\left|\frac{t+x}{t-x}\right|\mathrm{d}t + \frac{\mathrm{d}}{\mathrm{d}x}\int_c^1 g(u^2)\log\left|\frac{u+x}{u-x}\right|\mathrm{d}u + \frac{\mathrm{d}}{\mathrm{d}x}\int_0^b h(t)\mathrm{d}t\int_0^\infty \xi^{-1}M(\xi h)\sin(\xi t)\sin(\xi x)\mathrm{d}\xi$$
$$+ \frac{\mathrm{d}}{\mathrm{d}x}\int_c^1 g(u^2)\mathrm{d}u\int_0^\infty \xi^{-1}M(\xi h)\sin(\xi u)\sin(\xi x)\mathrm{d}\xi = \sqrt{\frac{\pi}{2}}\frac{p(x)}{\mu_1 s_1}, \quad 0 < x < b,\ c < x < 1. \tag{3.14}$$

Setting, $h(t) = h_0(t) + h^{-2}h_1(t) + 0(h^{-4})$ and $g(u^2) = g_0(u^2) + h^{-2}g_1(u^2) + 0(h^{-4})$, the integral equations in (3.14) reduce to

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_0^b h_0(t)\log\left|\frac{t+x}{t-x}\right|\mathrm{d}t + 2\int_c^1 \frac{ug_0(u^2)}{u^2-x^2}\mathrm{d}u = \sqrt{\frac{\pi}{2}}\frac{p(x)}{\mu_1 s_1} \tag{3.15}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_0^b h_1(t)\log\left|\frac{t+x}{t-x}\right|\mathrm{d}t + 2\int_c^1 \frac{ug_1(u^2)}{u^2-x^2}\mathrm{d}u$$
$$= -2P\int_0^b\left[h_0(t)\mathrm{d}t + \int_c^1 ug_0(u^2)\mathrm{d}u\right], \quad 0 < x < b,\ c < x < 1, \tag{3.16}$$

with

$$\int_c^1 g_i(u^2)\mathrm{d}u = 0,\ i = 0, 1, \tag{3.17}$$

where

$$P = \frac{-\mu_2 s_2}{2(\mu_1 s_1 + \mu_2 s_2)s_1^2} + \frac{\mu_2 s_2(\mu_2 s_2 - \mu_1 s_1)}{8(\mu_1 s_1 + \mu_2 s_2)^2 s_1^2}.$$

Rewriting Eq. (3.15) as

$$\int_0^b h_0(t)\log\left|\frac{t+x}{t-x}\right|\mathrm{d}t = \pi F_1(x) \tag{3.18}$$

where

$$F_1(x) = \frac{1}{\pi}\int_0^x\left[\sqrt{\frac{\pi}{2}}\frac{p(y)}{\mu_1 s_1} - \int_c^1 \frac{2ug_0(u^2)}{u^2-y^2}\mathrm{d}u\right]\mathrm{d}y$$

and using Cooke's result (1968), the solution to the integral equation in (3.18) is found to be

$$h_0(t) = -\frac{2}{\pi\mu_1 s_1}\frac{t}{\sqrt{b^2-t^2}}P_1(t) - \frac{2}{\pi}\frac{t}{\sqrt{b^2-t^2}}\int_c^1 \frac{\sqrt{u^2-b^2}g_0(u^2)\mathrm{d}u}{u^2-t^2} \tag{3.19}$$

where

$$P_1(t) = \frac{1}{\sqrt{2\pi}}\int_0^b \frac{\sqrt{b^2-x^2}}{x^2-t^2}p(x)\mathrm{d}x.$$

Then from Eq. (3.15) the integral equation for $g_0(u^2)$ is derived as

$$\int_c^1 \frac{\sqrt{u^2-b^2}g_0(u^2)}{u^2-x^2}\mathrm{d}u = F_2(x) \tag{3.20}$$

where

$$F_2(x) = \frac{\sqrt{x^2-b^2}}{x}\left[\sqrt{\frac{\pi}{2}}\frac{p(x)}{\mu_1 s_1} + \frac{2}{\pi\mu_1 s_1}\int_0^b \frac{t^2 P_1(t)}{\sqrt{b^2-t^2}(t^2-x^2)}\mathrm{d}t\right].$$

Now using Hilbert transform technique, the solution of the Eq. (3.20) is found to be

$$\begin{aligned} g_0(u^2) = {} & \frac{2u}{\pi\sqrt{(u^2-c^2)(1-u^2)(u^2-b^2)}}\int_c^1 \frac{\sqrt{x^2(x^2-c^2)(1-x^2)}}{x^2-u^2}F_2(x)\mathrm{d}x \\ & + \frac{uC_1}{\sqrt{(u^2-c^2)(1-u^2)(u^2-b^2)}} \end{aligned} \tag{3.21}$$

where C_1 is unknown constant to be determined from Eq. (3.17). Then closed form expression for $h_0(t)$ may be obtained from Eq. (3.19) when use of (3.21) is made there. Again applying the same procedure and using the above results, analytic expressions of $h_1(t)$ and $g_1(u^2)$ may also be derived. As a particular case of the problem, setting $p(x) = p$, a constant, analytical expressions for $h_j(t)$ and $g_j(u^2)$, $j = 0, 1$ are obtained as

$$h_0(t) = \sqrt{\frac{\pi}{2}}\frac{p}{\mu_1 s_1}\sqrt{\frac{t^2(c^2-t^2)}{(b^2-t^2)(1-t^2)}} + \frac{t\cdot C_1}{\sqrt{(b^2-t^2)(c^2-t^2)(1-t^2)}},$$

$$h_1(t) = -\frac{2PR}{\pi}\sqrt{\frac{t^2(c^2-t^2)}{(b^2-t^2)(1-t^2)}} - \frac{t\cdot C_2}{\sqrt{(b^2-t^2)(c^2-t^2)(1-t^2)}},$$

$$g_0(u^2) = \sqrt{\frac{\pi}{2}}\frac{p}{\mu_1 s_1}\sqrt{\frac{u^2(u^2-c^2)}{(u^2-b^2)(1-u^2)}} + \frac{u\cdot C_1}{\sqrt{(u^2-b^2)(u^2-c^2)(1-u^2)}},$$

$$g_1(u^2) = -\frac{2PR}{\pi}\sqrt{\frac{u^2(u^2-c^2)}{(u^2-b^2)(1-u^2)}} + \frac{u\cdot C_2}{\sqrt{(u^2-b^2)(u^2-c^2)(1-u^2)}},$$

where $R = \sqrt{\frac{\pi}{2}}\frac{p}{\mu_1 s_1}[I_0^b + I_c^1] - C_1[J_0^b - J_c^1]$,

$$I_m^n = \int_m^n \frac{t^2\sqrt{c^2-t^2}}{\sqrt{(b^2-t^2)(1-t^2)}}\mathrm{d}t,$$

$$J_m^n = \int_m^n \frac{t^2\mathrm{d}t}{\sqrt{(b^2-t^2)(c^2-t^2)(1-t^2)}},$$

$$C_j = A_j\left[(1-b^2)\frac{E}{F} - (c^2-b^2)\right],\ j = 1, 2$$

with

$$A_1 = \sqrt{\frac{\pi}{2}}\frac{p}{\mu_1 s_1},\ A_2 = \frac{2PR}{\pi}.$$

In the above $F = F(\pi/2, q)$ and $E = E(\pi/2, q)$ are the elliptic integrals of first and second kinds respectively and $q = \sqrt{\frac{1-c^2}{1-b^2}}$.

The stress intensity factors K_b, K_c and K_1 at the crack tips are found to be

$$K_b = p\sqrt{\frac{b(1-b^2)}{c^2-b^2}\frac{E}{F}}\left[1-\frac{2P}{\pi}\frac{M}{h^2}\right] + 0(h^{-4}), \tag{3.22}$$

$$K_c = p\sqrt{\frac{c}{(c^2-b^2)(1-c^2)}}\left[(1-b^2)\frac{E}{F}-(c^2-b^2)\right]\left[1-\frac{2P}{\pi}\frac{M}{h^2}\right] + 0(h^{-4}), \tag{3.23}$$

$$K_1 = p\sqrt{\frac{1-b^2}{1-c^2}}\left[1-\frac{E}{F}\right]\left[1-\frac{2P}{\pi}\frac{M}{h^2}\right] + 0(h^{-4}), \tag{3.24}$$

where $M = \left[I_0^b + I_c^1 + \left\{(1-b^2)\frac{E}{F}-(c^2-b^2)\right\}(J_0^b - J_c^1)\right]$.

The non-dimensional stress intensity magnification factors M_b, M_c and M_1 at the crack tips $x = b$, $x = c$ and $x = 1$ are defined as $M_b = K_b/K_b^*$, $M_c = K_c/K_c^*$ and $M_1 = K_1/K_1^*$ where K_b^* is the stress intensity factor at $x = b$ due to the presence of the central crack only (Dhaliwal, 1992) and K_c^*, K_1^* are the stress intensity factors at $x = c$, $x = 1$ respectively due to the presence of the outer cracks only (Das, 2003) and these are given by

$$K_b^* = p\sqrt{b}\frac{E}{F}\left[1-\frac{2P}{\pi}\frac{Q_1}{h^2}\right] + 0(h^{-4}) \quad \text{where} \quad Q_1 = I_0^b + (1-b^2)\left[\frac{E}{F}-1\right]J_0^b \tag{3.25}$$

$$K_c^* = \frac{p}{\sqrt{c(1-c^2)}}\left[\frac{E}{F}-c^2\right]\left[1-\frac{2P}{\pi}\frac{Q_2}{h^2}\right] + 0(h^{-4}), \tag{3.26}$$

$$K_1^* = \frac{p}{\sqrt{1-c^2}}\left[1-\frac{E}{F}\right]\left[1-\frac{2P}{\pi}\frac{Q_2}{h^2}\right] + 0(h^{-4}), \tag{3.27}$$

where

$$Q_2 = I_c^1 - \left[\frac{E}{F}-c^2\right]J_c^1.$$

4. Particular cases

(A) A particular case of the problem of two bonded dissimilar half planes ($h = \infty$) containing three Griffith cracks at their interface is being considered.
Here $M(\xi h) = 0$ as $h \to \infty$.
Hence (3.10) becomes

$$\int_c^\infty \xi f(\xi)\cos\xi x \mathrm{d}\xi = \sqrt{\frac{\pi}{2}}\frac{p(x)}{\mu_1 s_1}, \quad 0 \leqslant x \leqslant b, \ c \leqslant x \leqslant 1.$$

Proceeding in similar manner as in Section 3, we get the stress intensity factors as

$$K_b = p\sqrt{\frac{b(1-b^2)}{c^2-b^2}\frac{E}{F}},$$

$$K_c = p\sqrt{\frac{c}{(c^2-b^2)(1-c^2)}}\left[(1-b^2)\frac{E}{F}-(c^2-b^2)\right],$$

$$K_1 = p\sqrt{\frac{1-b^2}{1-c^2}}\left(1-\frac{E}{F}\right).$$

(B) When $1 - c =$ constant and $(1 + c) \to \infty$, then the problem reduces to a problem of a single Griffith crack situated at the interface of two bonded dissimilar elastic media under antiplane shear.

(C) If $b = 0$, the problem reduces to a problem of a pair of interfacial Griffith cracks bonded between two dissimilar elastic media under antiplane shear. Here the stress intensity factors at the tips of the cracks are same as the expressions of (3.26) and (3.27). These results are in complete agreement with the results of Das (2003).

5. Other possible boundary conditions

In Section 3, we deduced the expression of the stress intensity factors at the tips of the cracks when the layer boundary is stress free. Some other possible conditions have been considered below:

(A) If the layer is fixed, in this case the boundary condition (2.7) will be replaced by

$$\omega^{(1)}(x,h) = 0, \quad -\infty < x < \infty, \tag{5.1}$$

$$M(\xi h) = \frac{1-\tanh s_1\xi h}{\frac{\mu_1 s_1}{\mu_2 s_2} + \tanh s_1 \xi h}.$$

Accordingly, analysis and results of Section 3 remain valid subject to above modified value of $M(\xi h)$.

(B) In this case we will consider that the interfacial Griffith cracks between two bonded dissimilar stress free layers. Here the boundary conditions (2.4)–(2.7) hold with one additional boundary condition

$$\tau_{yz}^{(2)}(x,-h) = 0, \quad -\infty < x < \infty. \tag{5.2}$$

Here Eq. (3.10) becomes

$$\int_0^\infty \xi f(\xi)[1+M(\xi h)]\cos\xi x \mathrm{d}\varsigma = \sqrt{\frac{\pi}{2}}\left(1+\frac{\mu_1 s_1}{\mu_2 s_2}\right)\frac{p(x)}{\mu_1 s_1}, \quad 0<x<b,\ c<x<1 \tag{5.3}$$

where

$$M(\xi h) = \frac{2\left[-\mu_1 s_1 \mathrm{e}^{-2s_2\xi h} - \mu_2 s_2 \mathrm{e}^{-2s_1\xi h} + (\mu_2 s_2 + \mu_1 s_1)\mathrm{e}^{-2(s_1+s_2)\xi h}\right]}{(\mu_2 s_2+\mu_1 s_1)(1-\mathrm{e}^{-2(s_1+s_2)\xi h}) + (\mu_2 s_2 - \mu_1 s_2)(\mathrm{e}^{-2s_1\xi h} - \mathrm{e}^{-2s_2\xi h})}.$$

(C) In this case we will consider the problem of interfacial Griffith cracks between two bonded dissimilar fixed layers.Here boundary conditions of (2.7) and (5.2) will be replaced by the following boundary conditions:

$$\omega^{(1)}(x,h) = 0,\ -\infty < x < \infty, \tag{5.4}$$

$$\omega^{(2)}(x,-h) = 0,\ -\infty < x < \infty. \tag{5.5}$$

Hence Eq. (5.3) will be replaced by

$$\int_0^\infty \xi f(\xi)[1+M(\xi h)]\cos\xi x \mathrm{d}\xi = \sqrt{\frac{\pi}{2}}\left(1-\frac{\mu_1 s_1}{\mu_2 s_2}\right)\frac{p(x)}{\mu_1 s_1}$$

where

$$M(\xi h) = 2\left[\frac{\mu_2 s_2 \mathrm{e}^{-2s_1\xi h} - 2\mu_1 s_1 \mathrm{e}^{-2s_2\xi h} + (\mu_2 s_2 - \mu_1 s_1)\mathrm{e}^{-2(s_1+s_2)\xi h}}{(\mu_2 s_2 - \mu_1 s_1)(1-\mathrm{e}^{-2(s_1+s_2)\xi h}) - (\mu_2 s_2+\mu_1 s_1)(\mathrm{e}^{-2s_1\xi h} - \mathrm{e}^{-2s_2\xi h})}\right].$$

6. Numerical results and discussion

In this section numerical results of the stress intensity magnification factors for various values of crack length and crack speed are presented through the Figs. 2–10 when the layer thickness $h = 4(2)12$, the non-dimensional crack tip velocity $v/k_1 = 0.2$, 0.6, 0.9 and the ratio of the material constants $k_1/k_2 = 0.6$ and $\mu_1/\mu_2 = 0.5$. For studying the interactions between the central and the external cracks, plots of stress intensity magnification factors through Figs. 2–10 have been made. It is observed from Figs. 2–4 that on keeping the central crack length fixed at $b = 0.5$, the stress intensity magnification factor M_b decreases with a decrease in the outer crack length and increases with an increase in v/b_1. In this case the interaction effect of outer crack on the central crack is a mixture of amplification and shielding. When the outer crack is relatively smaller

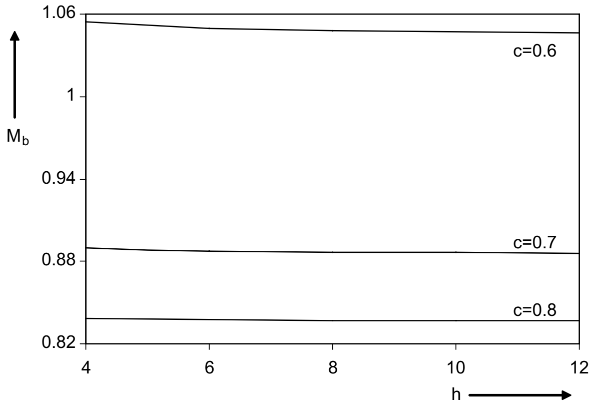

Fig. 2. Plot of M_b versus h at $b = 0.5$ and $v/k_1 = 0.2$.

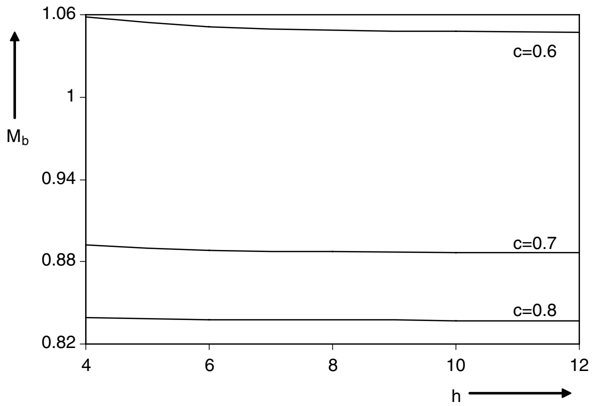

Fig. 3. Plot of M_b versus h at $b = 0.5$ and $v/k_1 = 0.6$.

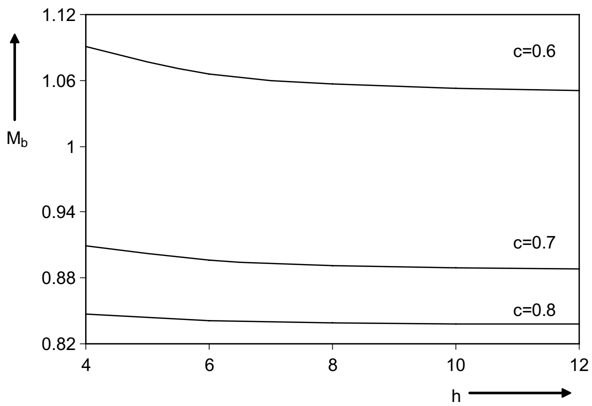

Fig. 4. Plot of M_b versus h at $b = 0.5$ and $v/k_1 = 0.9$.

($c = 0.7$, 0.8) and the normalized crack tip velocity is equal to $v/k_1 = 0.2$, the shielding effect (since M_b is less than unity, the interaction effect is one of shielding) is maximum. It is also observed that the shielding effect increases slowly with the increase in the depth of the layer. As the outer crack length increases, i.e., the outer crack comes closer to central crack ($c = 0.6$), the effect of interaction is that of amplification ($M_b > 1$) which again diminishes with the increase in h and increases with the increase in v/k_1.

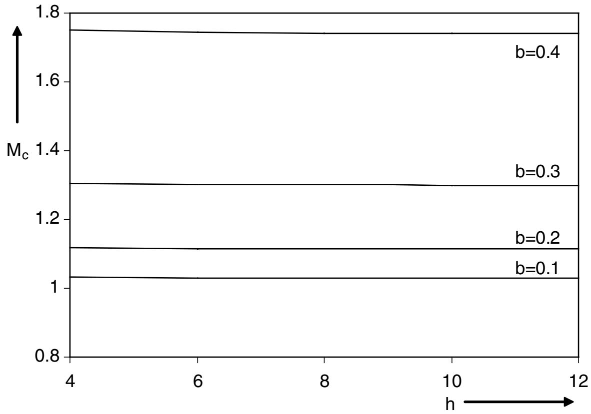

Fig. 5. Plot of M_c versus h at $c = 0.5$ and $v/k_1 = 0.2$.

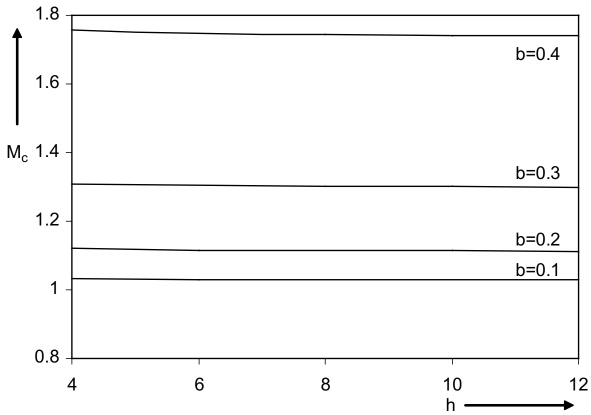

Fig. 6. Plot of M_c versus h at $c = 0.5$ and $v/k_1 = 0.6$.

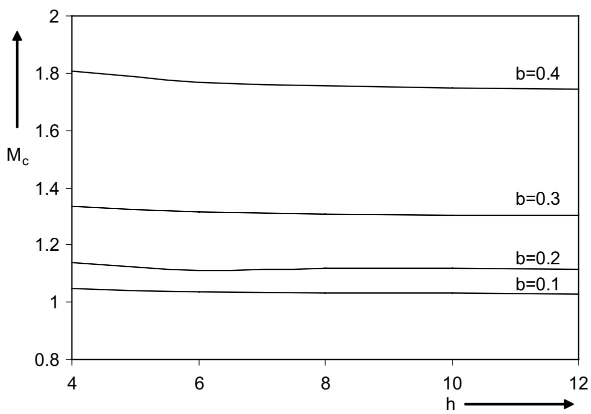

Fig. 7. Plot of M_c versus h at $c = 0.5$ and $v/k_1 = 0.9$.

When the outer crack length is kept fixed at $c = 0.5$, it is observed from Figs. 5–10 that the stress intensity magnification factors M_c and M_1 increase with an increase in the central crack length, with an increase in v/b_1

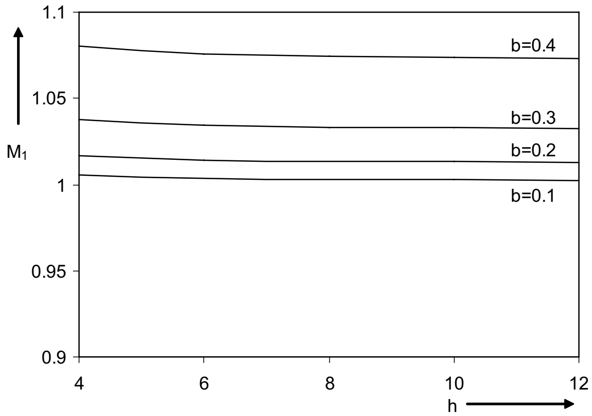

Fig. 8. Plot of M_1 versus h at $c = 0.5$ and $v/k_1 = 0.2$.

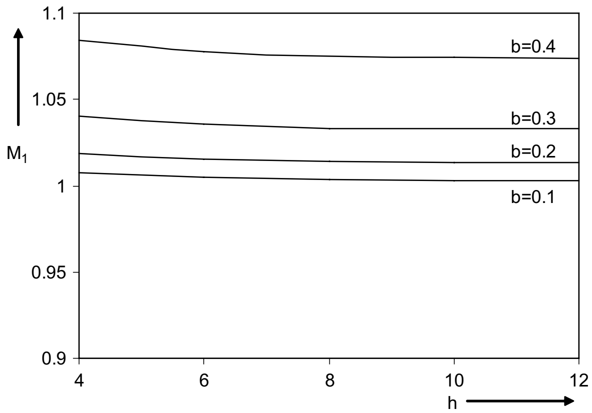

Fig. 9. Plot of M_1 versus h at $c = 0.5$ and $v/k_1 = 0.6$.

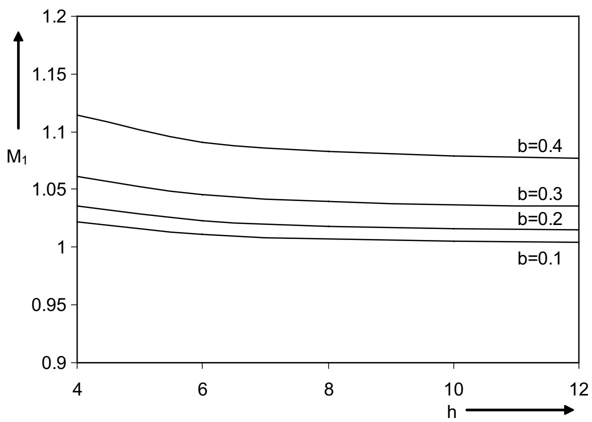

Fig. 10. Plot of M_1 versus h at $c = 0.5$ and $v/k_1 = 0.9$.

and decreases with the increase in h. In this case the interaction effects of the central crack on the outer crack are amplification only. At both the ends of the outer crack, interaction effect increases and the maximum amplification attains when the central crack tip is closer to the outer one ($b = 0.4$).

7. Conclusion

Thus we have seen that the effect of interaction between the central and outer cracks is a mixture of amplification and shielding or simply amplification depending on the length of the cracks, the crack speed and also the depth of the layer. When the outer crack is smaller and crack speed is less than unity, there is possible crack arrest of the central crack. When the outer crack is broad, there is propagation tendency of the central crack towards the outer crack. Again as the central crack length extends, the propagation tendency of outer crack at both ends increase due to increase of amplification.

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